Goal: Compute arc length and average value for curves.

Arc Length:

For a curve $y = f(x), a \leq x \leq b$ where $f'(x)$ is continuous, the length of the curve is given by:

For a curve $x = f(y), a \leq y \leq b$ where $f'(y)$ is continuous, the length of the curve is given by:

1. Find the exact length of the curve given by $y = \dfrac{1}{2}x^2, 0 \leq x \leq 2$.

2. Find the exact length of the curve given by $x = y^{3/2}$, $0 \leq y \leq 1$.

3. Find the exact length of the curve given by $y = \ln\big(\sec(x)\big)$, $0 \leq x \leq \pi/4$.

Average Value: The average value of a function f on the interval $[a, b]$ is given by:

4. Find the average value of the function $f(x) = 4x - x^2$ on the interval $[0, 4]$.

5. Find the average value of the function $g(y) = \sqrt[3]{x}$ on the interval $[1, 8]$.

6. Find the average value of the function $h(x) = \big(\cos(x)\big)^4 \sin(x)$ on the interval $[0, \pi]$.

Mean Value Theorem for Integrals: If f is continuous on $[a,b]$, then there exists a number c in $[a,b]$ such that

7. Let $f(x) = (x-3)^2$. Find c such that the average value of f is equal to $f(c)$ on the interval $[2,5]$.

8. Let $g(x) = \ln(x)$. Find c such that the average value of g is equal to $g(c)$ on the interval $[1, 3]$.

9. If f is continuous as $\displaystyle\int_1^3 f(x)\,dx = 8$, show that f takes on the value 4 at least once in the interval $[1, 3]$.

10. Here are some more problems to build your arc length and average value skills!

   (i) Find the length of $x = \cos(y)$ from $y = 0$ to $y = \pi$.

   (ii) Find the length of $y = \arccos(x)$ from $x = -1$ to $x = 1$.

   (iii) The velocity (ft/s) of an object at t seconds is given by $v(t) = t^3 - 3\ln(t+1) + 1$. Find the average velocity of the object during the first second of motion.

(iv) ♨ The single share price of stock in ATMOS is given by $p(t) = 4\sin(3t) + \dfrac{t^3}{2} - 2t^2 + 40$ where t is the number of days after April 5th. Find the average price of ATMOS stock from April 5th to April 9th.

(v) ♨ Prove the Mean Value Theorem for Integrals by applying the Mean Value Theorem for derivatives to the function $F(x) = \displaystyle\int_a^x f(t)\,dt$.